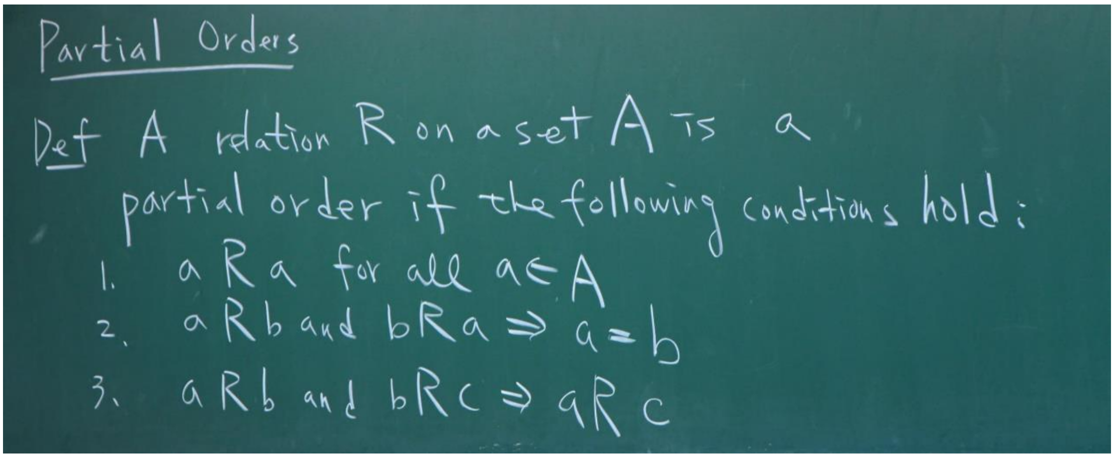
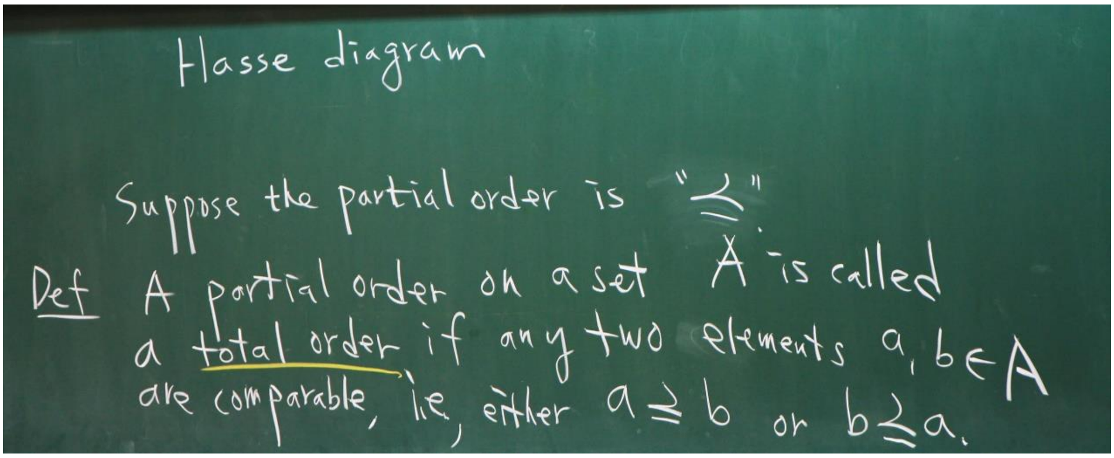
【10920 趙啟超教授離散數學 / 第 8 堂版書】





Partial Orders

Def A relation R on a set A is a partial order if the following conditions hold:

1. aRa for all $a \in A$
2. aRb and $bRa \Rightarrow a = b$
3. aRb and $bRc \Rightarrow aRc$

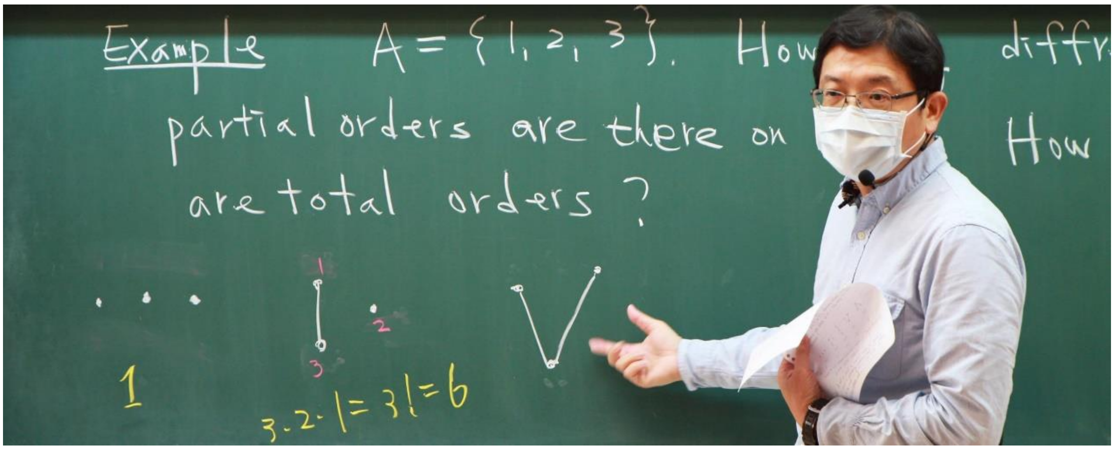
Hasse diagram

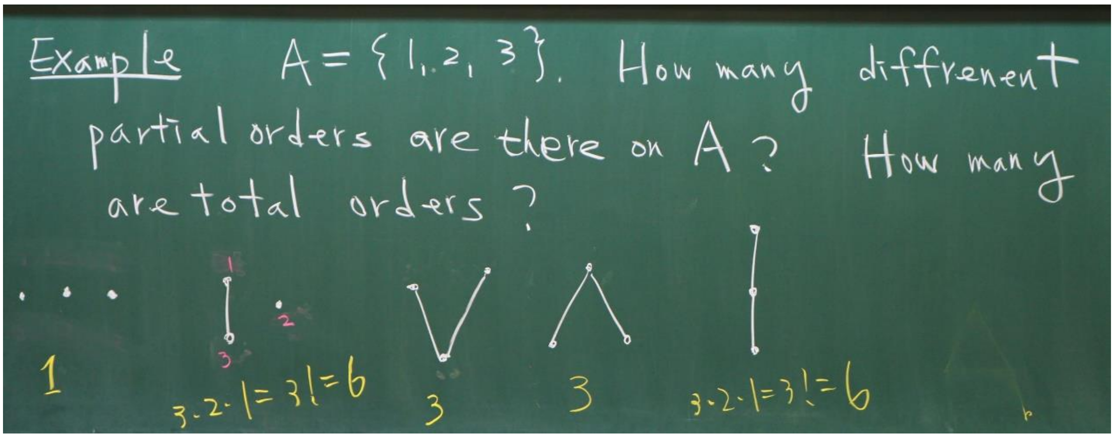
Suppose the partial order is "$\precsim$"

Def A partial order on a set A is called a total order if any two elements $a, b \in A$ are comparable, i.e., either $a \precsim b$ or $b \precsim a$.

Example "$\leq$" (less than or equal to) on the set $\mathbb{Z}$ is a total order.

Example Hasse diagram for a total order


Example A = {1, 2, 3}. How
partial orders are there on
How
are total orders ?
1
3·2·1 = 3! = 6


Example A = {1, 2, 3}. How many diffrenent
partial orders are there on A ? How many
are total orders ?
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3
3
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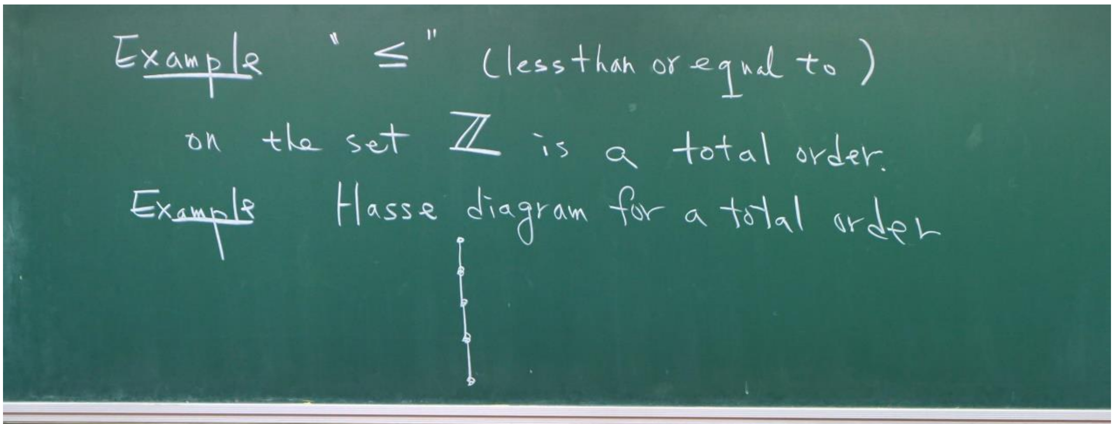
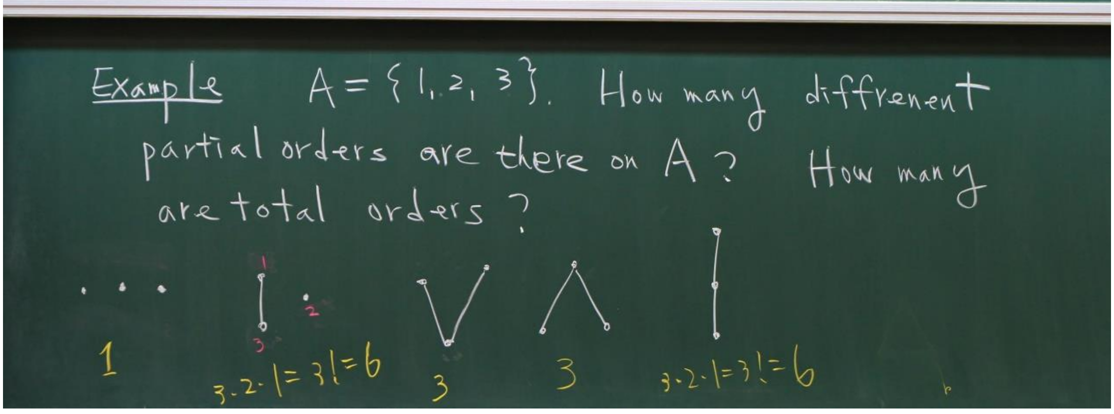

Example "≤" (less than or equal to)
on the set ℤ is a total order.
Example Hasse diagram for a total order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Total # of different partial orders

$= 1+6+3+3+6 = 19$

6 are total orders.

Def A set on which a partial order is defined is called a partially ordered set or poset. If the order is total, we have a totally ordered set.

Def Consider a patial order $\preceq$ on a set A.
We say that x is a maximal element in A if there is no $a \in A$ such that $x \prec a$.
Similarly, y is a minimal element in A if there is no $b \in A$ such that $b \prec y$.

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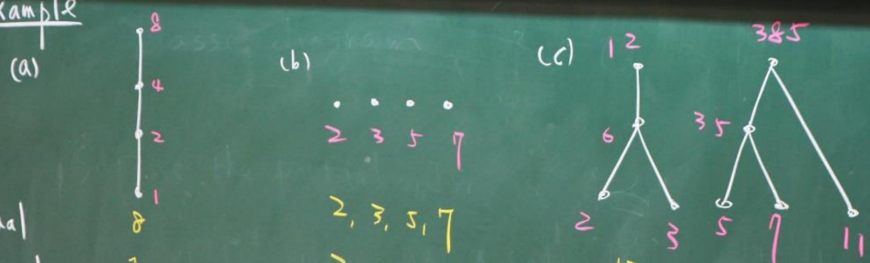
Def We say that x is a greatest element in A such that $a \preceq x$ for all $a \in A$. Similarly, y is a least element in A such that $y \preceq a$ for all $a \in A$.

Example



	(a)	(b)	(c)
maximal	8	2, 3, 5, 7	12, 385
minimal	1	2, 3, 5, 7	2, 3, 5, 7, 11
greatest	8	none	none
least	1	none	none

partial orders are there on A? How many are total orders?

1 $\quad 3\cdot2\cdot1=3!=6 \quad$ 3 $\quad$ 3 $\quad 3\cdot2\cdot1=3!=6$

Def Consider a partial order $\precsim$ on A with $B \subseteq A$. A element $x \in A$ is called a lower bound of B if $x \precsim b$ for all $b \in B$. Similarly, an element $y \in A$ is an upper bound of B if $b \precsim y$ for all $b \in B$.

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Def A element $x' \in A$ is called a greatest lower bound of B if x' is a lower bound of B and $x'' \precsim x'$ for all other lower bounds x'' of B. Similarly, an element $y' \in A$ is called a least upper bound of B if y' is an upper bound of B and $y' \precsim y''$ for all other upper bounds y'' of B.

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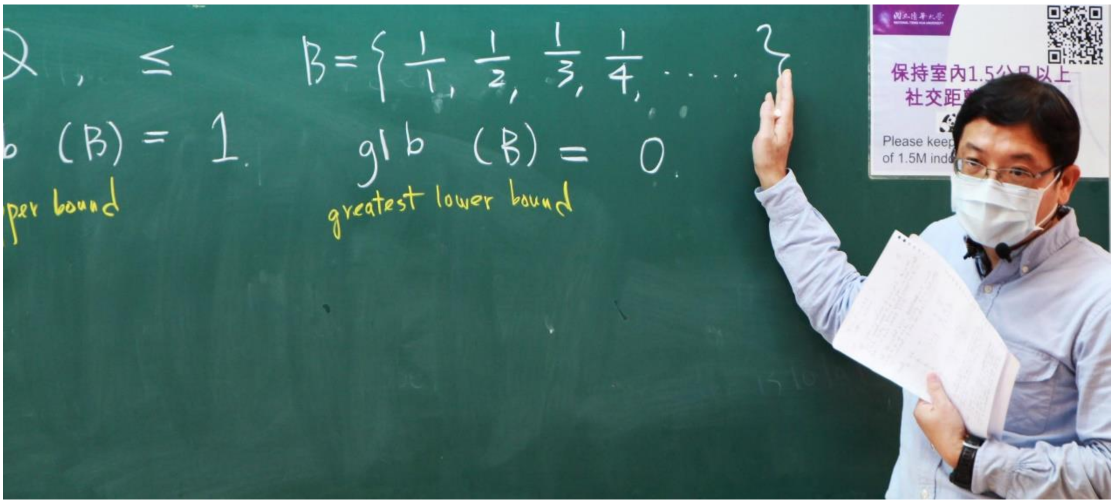
Q , ≤   B = { 1/1, 1/2, 1/3, 1/4, ... }
(B) = 1.   glb (B) = 0.
per bound   greatest lower bound
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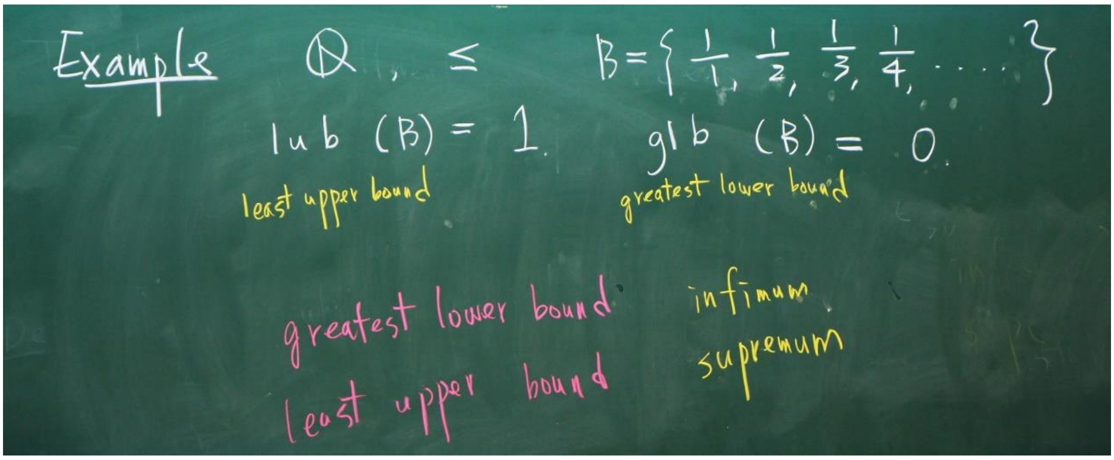
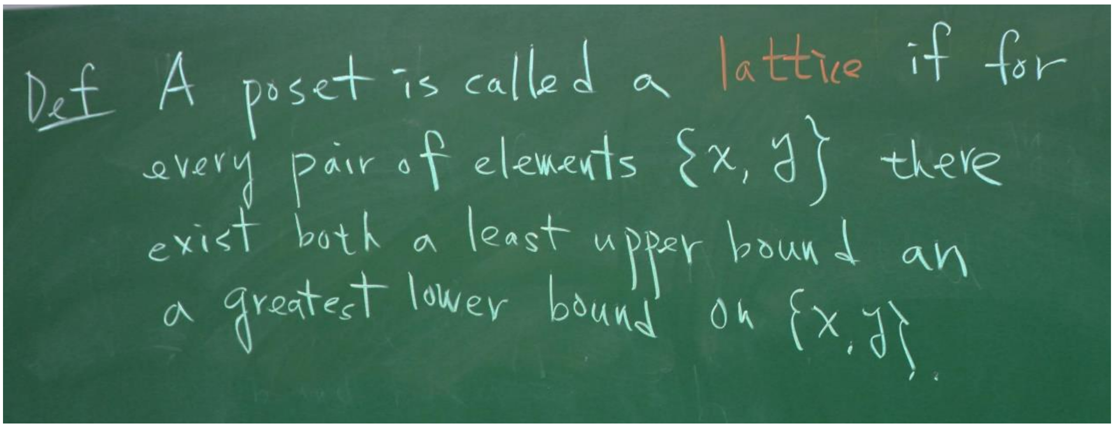
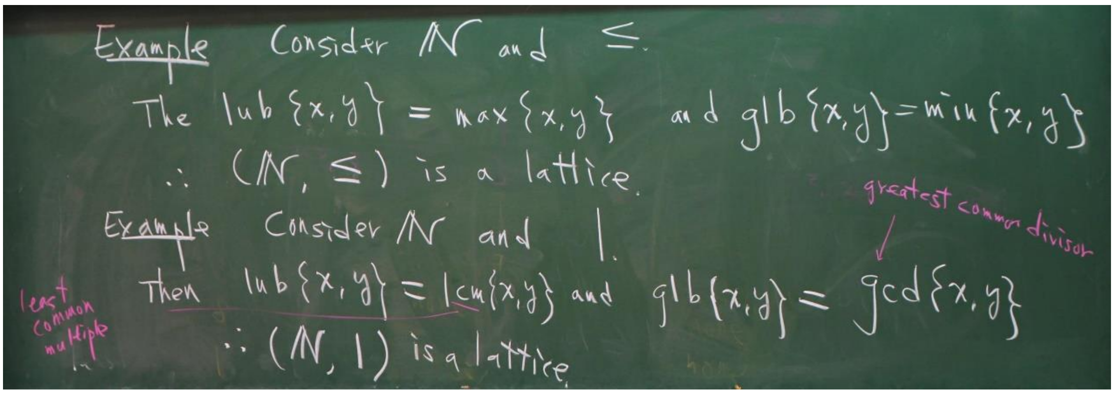
社交距離
Please keep
of 1.5M ind


Example Q , ≤   B = { 1/1, 1/2, 1/3, 1/4, ... }
lub (B) = 1.   glb (B) = 0.
least upper bound   greatest lower bound
greatest lower bound   infimum
least upper bound   supremum


Def A poset is called a lattice if for
every pair of elements {x, y} there
exist both a least upper bound an
a greatest lower bound on {x, y}.


Example Consider N and ≤.
The lub{x,y} = max{x,y} and glb{x,y} = min{x,y}
∴ (N, ≤) is a lattice.
Example Consider N and |.
greatest common divisor
Then lub{x,y} = lcm{x,y} and glb{x,y} = gcd{x,y}
least common multiple
∴ (N, |) is a lattice